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COMMENT

On an integral with modified Bessel function

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Abstract. The definite integral over $I_0(z)$ evaluated by Bakulev is shown to be a special case of a tabulated integral.

In a recent paper having this title Bakulev [1] went through considerable difficulty to evaluate the integral (slightly rewritten)

$$\int_0^a I_0(\sqrt{a^2-x^2})\cosh(Ax)\,\mathrm{d}x = \frac{\sinh(a\sqrt{1+A^2})}{\sqrt{1+A^2}}. \tag{1}$$

The purpose of this comment is to point out that equation (1), its generalization

$$\int_0^a (a^2-x^2)^{\nu/2} I_\nu(\sqrt{a^2-x^2})\cosh(Ax)\,\mathrm{d}x = \sqrt{\frac{2a}{\pi}}\,a^\nu \frac{I_{\nu+1/2}(a\sqrt{1+A^2})}{(\sqrt{1+A^2})^{\nu+1/2}} \tag{2}$$

together with a large number of others, which appear to be new, can actually be found in standard tables if one realizes that $\cosh(z)$, $\cos(z)$, $J_\nu(z)$ and $I_\nu(z)$, etc, are the same functions under the imaginary scale transformation $z \to \mathrm{i}z$. For example,

$$J_\nu(\mathrm{i}z) = i^\nu I_\nu(z). \tag{3}$$

Thus, (2) is the analytic continuation of equation (1.13) (50) of [2] with $b \to \mathrm{i}$ and $y \to \mathrm{i}A$.

References

[1] Bakulev A P 1991 *J. Phys. A: Math. Gen.* **24** 5747

[2] Erdelyi A *et al* 1954 *Tables of Integral Transforms* vol 1 (New York: McGraw Hill)

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